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Effect of Subcarrier Activation Ratio on the Performance of OFDM-IM over Rayleigh Fading Channel

Abdulrahman Ikram Siddiq, *Member, IEEE*

***Abstract*—Orthogonal Frequency Division Multiplexing with Index Modulation (OFDM-IM) is acquiring increasing attention due to its advantages over classical OFDM. These advantages include superior BER performance and transmit power saving since only a subset of the subcarriers is activated to transmit M-QAM symbols and the remaining subcarriers are turned off. The vast majority of the literature studies the OFDM-IM by assuming certain subcarrier activation ratios, r, together with other system parameters without figuring out the possible variation in system performance at other r values. In this paper, the effect of r and M on the spectral efficiency (SE) and BER of OFDM-IM is investigated. An inequality is developed to maximize SE while power saving is maintained. Moreover, the BER performance of an OFDM-IM system exploiting a maximum subcarrier energy detector over multipath Rayleigh channel is analyzed. An approximate average BER expression is derived and the effect of r and M is discussed. Computer simulation has showed the validity of the derived expressions.**

***Index Terms*—Orthogonal frequency division multiplexing, subcarrier index modulation, maximum energy detector, spectral efficiency, bit error rate.**

I. Introduction

ORTHOGONAL Frequency Division Multiplexing with Index Modulation (OFDM-IM) is a modification of the ordinary OFDM. It is the result of applying the concept of spatial modulation to the subcarriers of the OFDM block. The idea is to activate a subset of the subcarriers to carry M-ary constellation symbols while the rest are turned off. The subcarrier activation patterns (SAPs) are determined according to the data, therefore they are also used to convey information. This in general saves transmitted power and enables better BER performance with respect to classical OFDM [1], [2].

The idea of activating a subset of all available subcarriers was introduced in [3] for the sake of peak-to-average power ratio reduction. Next, the Subcarrier Index Modulation OFDM is presented in [4]. It proposed that data is split into two streams. One data stream is segmented and used to activate the subcarriers through a one-to-one relation between the majority bit value (1 or 0) with the corresponding subcarriers. The second data stream is used to modulate the active subcarriers. The system is reported to be able to achieve BER improvement over equivalent classical OFDM [4]. The disadvantages of this system are the need to send side information and error propagation. Next in [5], the system is modified to what is known then in the literature as OFDM-IM [1]. Since then, many works have been made to explore different OFDM-IM issues including BER performance, spectral efficiency (SE) improvement, detection scheme, application in MIMO systems, etc. That is, in [6] a tight approximate BER expression is derived. Different ways of subblock interleaving are used to improve: the Euclidean distance between received symbols [7], the achievable rate [8], [9], the SE and energy efficiency (EE) [10] and BER performance as in [11]. The use of a variable number of active subcarriers per subblock is proposed in [12] to improve the SE. It applied OFDM-IM to the in-phase and quadrature components independently. It has a BER performance loss with respect to classical OFDM-IM. The detector complexity is analyzed and low complexity alternatives are proposed in [13]-[16]. The performance of MIMO systems exploiting OFDM-IM is addressed in [17] and [18]. However, a common issue in most of the literature of OFDM-IM is that usually specific subblock size, n, number of active subcarriers, k, and M values are used. The effect of other values on the behavior of the system is not mentioned. Exceptionally, [2] and [10] involved analysis to determine k to optimize SE and EE. In [2], a scheme for maximizing EE by optimal subcarrier grouping and optimal selection of k is proposed. In [10], a subcarrier modulation selection scheme is proposed to improve EE without deteriorating SE. The SE is described as an optimal value problem and solved to find k that maximizes SE.

In this paper, the joint effect of the subcarrier activation ratio, r $(= k/n)$, the absolute value of n, and the symbol modulation order, M, on the energy saving (ES), SE and BER performance of OFDM-IM system using a maximum energy (ME) detector working over Rayleigh channel, is investigated. The aim is to develop a simple expression that can be used to expect the effect of given parameter values (r, n and M) on system performance, and/or to find parameter values (system operating point) that can achieve a required system performance.

II. OFDM-IM System Model

Basically, in OFDM-IM, the N-subcarrier OFDM block is divided into g independent subblocks, each consisting of n subcarriers. At each subblock, k out of these n subcarriers are activated to be modulated by M-ary data symbols. Then, the subcarrier activation ratio $r \in (0, 1]$. The number of all possible SAPs is C_k^n $(= n!/(n-k)!k!)$. The selection of

A. I. Siddiq is with the Department of Electronic and Control Engineering, Northern Technical Univ., Technical College, Kirkuk, Iraq.

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a specific SAP is made by mapping p_1 $(= \lfloor log_2 C_k^n \rfloor)$ bits of data according to a predefined look-up table, where $\lfloor . \rfloor$ is the greatest lower integer function. Then, the actual number of used SAPs is 2^{p_1} which is equal to the greatest lower integer power of 2 of C_k^n since $C_k^n > 2^{p_1}, \forall\, n$ and $k > 1$. Moreover, p_2 $(= k log_2 M)$ bits are modulated onto the k active subcarriers of a subblock. Then, p $(= p_1 + p_2)$ bits are loaded to a subblock, and a total of $m = gp$ bits are conveyed by transmitting one OFDM block. Afterwards, the resulting OFDM-IM frequency domain block, $\mathbf{x} = [x_0, x_1, ..., x_{N-1}]^T$ undergoes the operations of cyclic prefix (CP) addition and IFFT as in ordinary OFDM transmitter. The time domain block is transmitted over a slowly time-varying multipath Rayleigh fading channel. The fading channel coefficients are assumed to be quasi-static. That is, they are fixed during the period of one block. At the receiver, CP is removed and an N-point FFT is applied to recover a replica of $\mathbf{x}$. The equivalent frequency domain output of the channel can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w} \tag{1}$$

where $\mathbf{y} = [y_0, y_1, ..., y_{N-1}]^T$, $\mathbf{H} = diag(h_0, h_1, ..., h_{N-1})$, h_i is the channel fading coefficient with $h_i \sim \mathcal{CN}(0, 1)$ and $\mathbf{w} = [w_0, w_1, ..., w_{N-1}]^T$ is an independent additive white Gaussian noise (AWGN) such that $w_i \sim \mathcal{CN}(0, N_0)$.

III. Effect of r on ES and SE

A. Energy Saving

Perhaps the main motivation behind the concept of index modulation is the attempt to optimize power consumption [5]. For a given M-QAM symbol constellation, let E_s be the average symbol energy. Then, the total energy of an N-subcarrier block in classical OFDM and OFDM-IM are $E_{OFDM} = NE_s$ and $E_{IM} = gkE_s = rNE_s$, respectively. Therefore, for the same N and M, OFDM-IM achieves a saving factor of $(1 - r)$ in transmit signal power with respect to OFDM. The amount of saved energy linearly depends on r and it is independent on the absolute value of n. At the extreme case when r is unity, OFDM-IM turns back to OFDM and E_{IM} is maximum $(= E_{OFDM})$. Whereas as r gets away from unity, E_{IM} decreases. That is, more energy is saved in OFDM-IM at lower M and/or r values and vice versa.

B. Spectral Efficiency

The number of transmitted bits per subcarrier is the SE measured in b/s/Hz. It is $log_2 M$ for OFDM and p/n for OFDM-IM, denoted by SE_{OFDM} and SE_{IM}, respectively. The former depends on the modulation order and it does not depend on N as long as CP is not considered. When CP is used, both SE_{OFDM} and SE_{IM} are scaled by a reduction factor of $N/(N + L)$, where L is the length of CP. For simplicity and without loss of generality, CP is ignored in this analysis. However, SE_{IM} can be written as

$$SE_{IM} = \frac{1}{n} \lfloor log_2 C_k^n \rfloor + r\, log_2 M \tag{2}$$

Proper selection of k, n, and M can maximize SE_{IM}. That is, the second term of (2) directly proportional with r and M. This characterizes the SE-ES trade-off, since by using higher r and M values, more energy is spent for higher SE. The first term of (2) depends directly on $\lfloor log_2 C_k^n \rfloor$ which in turn depends on the number of used SAPs, and reversely on the absolute value of n. The greatest value of C_k^n is when $r = 0.5$ and it decreases symmetrically as r goes to its extreme values. For all r, $\lfloor log_2 C_k^n \rfloor / n$ increases with n since the rate of growth of $\lfloor log_2 C_k^n \rfloor$ with n is faster than the denominator, n. Therefore, for given r and M, SE_{IM} increases with n. Mathematically, for SE_{IM} to be greater or equal to SE_{OFDM}, then the following inequality should apply

$$\frac{1}{n} \lfloor log_2 C_k^n \rfloor \geq log_2 M - r\, log_2 M \tag{3}$$

The term $\lfloor log_2 C_k^n \rfloor$ can be rewritten as $log_2 |\lfloor C_k^n \rfloor|$, where $|\lfloor . \rfloor|$ is used to represent the approximation to the greatest lower integer power of 2. Then, (3) becomes

$$|\lfloor C_k^n \rfloor| \geq M^{n-k} \tag{4}$$

The following cases can be observed

- When $r = 1$ $(k = n)$, then for all n and M, both sides of (4) are 1, and OFDM-IM is identical to OFDM. Referring to (2), $SE_{IM} = SE_{OFDM} = log_2 M$.
- When r is minimum, which occurs at minimum k for a given n. The smallest physical k is unity. Then, for all n and M, the rhs of (4), M^{n-1}, is greater than the lhs, $|\lfloor n \rfloor|$, which violates (4). This makes $SE_{IM} < SE_{OFDM}$ with an exception at $n = 2$ and $M = 2$ where $SE_{IM} = SE_{OFDM}$. For a given M $(M > 2)$ and $r = 1/n$, in the limit, SE_{IM} vanishes as n goes to infinity, since $\lim_{n \longrightarrow \infty} (\lfloor log_2 n \rfloor + log_2 M)/n = 0$.
- When $r = 0.5$ $(k = n/2)$, then (4) is reduced to $|\lfloor C_{n/2}^n \rfloor| \geq M^{n/2}$. It can be shown that $M^{n/2}$ is greater than $|\lfloor C_{n/2}^n \rfloor|$ for all n and $M > 2$, and hence $SE_{IM} < SE_{OFDM}$, with SE_{IM} asymptotically approaches SE_{OFDM} at large n. Except for $M = 2$, (4) is valid for all n and $SE_{IM} \geq SE_{OFDM}$.
- When r is maximum, which occurs at maximum k for a given n. The largest physical k is $n - 1$. In this case, (4) becomes $|\lfloor n \rfloor| \geq M$. Then for all M and $n \geq M$, we have $SE_{IM} \geq SE_{OFDM}$. In this case, r approaches unity at large n, and hence SE_{IM} approaches SE_{OFDM} since $\lim_{n \longrightarrow \infty} (\lfloor log_2 n \rfloor / n + r log_2 M) = log_2 M$.

In Fig. 1, SE_{IM} and SE_{OFDM} are plotted with n for $M = 4$ and $M = 16$ and $r = 1/n$, 0.5, and $(n - 1)/n$. The plots emphasize the above discussion. Generally, any other (r, n, M) combination other than the discussed values, that make (4) valid can maximize SE_{IM} to the level of SE_{OFDM} or even greater. The very interesting and most important issue to note is that SE_{IM} can exceed SE_{OFDM} at certain r, n, and M values with the energy saving advantage is not lost.

IV. BER Derivation

In this section, an approximate analytic expression is derived for the BER of OFDM-IM system over multipath Rayleigh fading channel. The ME detector is used due to its simplicity with respect to the maximum likelihood and logarithmic likelihood ratio detectors [14], [15].

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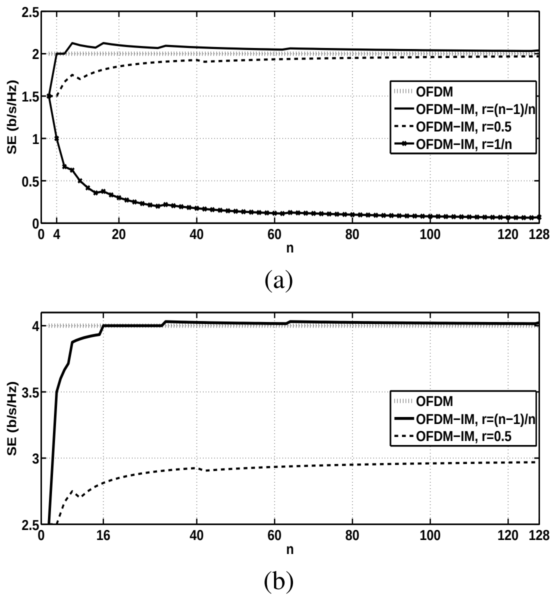

Fig. 1. Effect of r and n on SE for (a) $M = 4$ and (b) $M = 16$

For a received subblock, the receiver first detects the associated SAP and maps it to p_1 bits. Then, QAM demodulation is applied to the active subcarriers independently to determine p_2. SAP detection is performed by considering the k highest energy subcarriers as active. This leads to better carrier state detection performance as compared to simple thresholding [5]. Most importantly, this SAP detection scheme discards $(2^n - C_k^n)$ invalid SAPs from being detected and ensures that a received SAP always belongs to the space of C_k^n patterns of k active subcarriers.

However, a detected SAP could be identical to the transmitted SAP, correct, or incorrect being one of the remaining $C_k^n - 1$ SAPs. In the case of incorrect SAP detection, whether the detected SAP is used or unused by the system, the QAM demodulation stage will fail to estimate the transmitted p_2 bits correctly. This is because of the demodulation of some unmodulated subcarriers. Therefore, p_1 and p_2 are incorrect when a SAP is wrongly detected. On the other hand, if the detected SAP is correct, then, p_1 is correct but there is still a possibility of error in p_2. That is, if the QAM demodulation of one or more of the received active subcarriers was erroneous, then p_2 is affected. Otherwise, an error-free QAM demodulation for the active subcarriers of a correctly detected SAP results in receiving p_1 and p_2 correctly.

The probabilities of these different SAP detection cases are evaluated as follows. The estimation process of the activity of a single subcarrier is similar to coherent binary ASK detection whose probability of error over Rayleigh fading channel is given in [19] as

$$P_{BASK} = 0.5\left(1 - \sqrt{\frac{\gamma_b}{1+\gamma_b}}\right) \tag{5}$$

where $\gamma_b = \alpha^2 \frac{E_b}{N_o}$ is the average SNR per bit, and α and E_b are the average fading amplitude and the average bit energy, respectively. To have d errors in a received SAP, this means that there are d simultaneous errors in detecting the activity of the subcarriers in the received subblock. Fortunately, for given n and k, the number of different subcarrier states between any transmitted SAP$_i$ and an incorrectly received SAP$_j$ is known and can be calculated using

$$d_{i,j} = \sum_{\lambda=1}^{n} (SAP_{i,\lambda} \oplus SAP_{j,\lambda}) \tag{6}$$

where $1 \leq i \leq 2^{p_1}$, $1 \leq j, j \neq i \leq C_k^n$, λ is a dummy subcarrier index and $\oplus$ is the exclusive NOR operator. Therefore, the probability of detecting an incorrect SAP with $d_{i,j}$ subcarrier activity detection errors is

$$P_{ICi,j} = P_{BASK}^{d_{i,j}} \tag{7}$$

Then, the average probability of detecting incorrect (used or unused) SAPs is given as

$$P_{IC} = \frac{1}{2^{p_1}(C_k^n - 1)} \sum_{i=1}^{2^{p_1}} \sum_{j=1, j\neq i}^{C_k^n} P_{ICi,j} \tag{8}$$

For equiprobable data bits, such a received SAP causes an average error of about $0.5p_1$ in the demapped p_1 bits [15]. In addition, at high SNR, the process of demodulating M-QAM Gray encoded symbols from the detected SAP results mostly in 1 bit error per symbol as described in [5] and [15]. Having k such symbols, then k bits out of p are exposed to this error possibility. The BER contribution of an incorrect received SAP is

$$B_{IC} = P_{IC}\left(\frac{0.5p_1 + k}{p}\right) \tag{9}$$

Next, the second BER possibility occurs due to demodulation errors of the active subcarriers within a correctly received SAP. The approximate average bit error probability, P_b, of M-QAM demodulation over Rayleigh fading channel is given in [4] as

$$P_b = \frac{2}{log_2 M}\left(\frac{\sqrt{M}-1}{\sqrt{M}}\right)\left(1 - \sqrt{\frac{1.5\gamma_s}{M-1+1.5\gamma_s}}\right) - \left(\frac{\sqrt{M}-1}{\sqrt{M}}\right)^2 \times \frac{1}{log_2 M}\left[1 - \sqrt{\frac{1.5\gamma_s}{M-1+1.5\gamma_s}}\left(\frac{4}{\pi}tan^{-1}\sqrt{\frac{M-1+1.5\gamma_s}{1.5\gamma_s}}\right)\right] \tag{10}$$

where $\gamma_s (= \gamma_b \, log_2 M)$ is the average SNR per symbol. However, P_b occurs at each one of the k active subcarriers independently. This error affects p_2 bits out of p bits conveyed by the received subblock. Then, the BER when a SAP is correctly detected is

$$B_C = (1 - P_{IC}) P_b \frac{kp_2}{p} \tag{11}$$

The total approximate average BER of OFDM-IM over Rayleigh fading channel is the sum of (9) and (11), given as

$$B_{OFDM-IM} = B_{IC} + B_C \tag{12}$$

The first term of (12) is mainly affected by the subcarrier activation ratio, r. Referring to (8), it can be shown that the larger is the set of SAPs ($r \to 0.5$), the smaller is P_{IC} and hence B_{IC}. Whereas, B_{IC} increases as r goes to 0 or 1. P_{IC} also depends on P_{BASK} which decays as SNR

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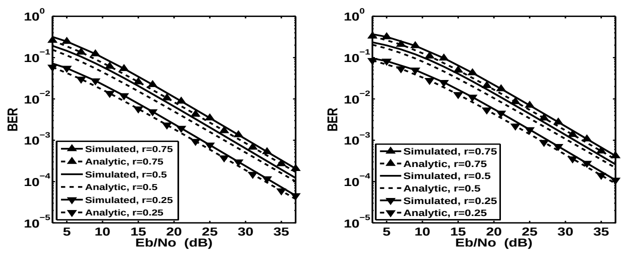
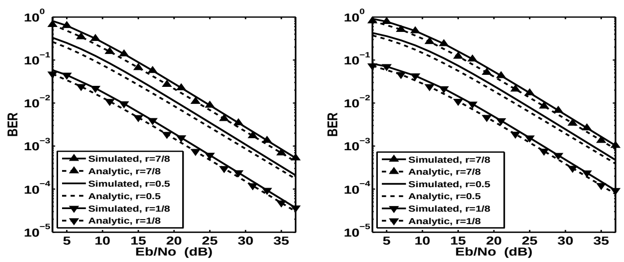


(a) (b)

Fig. 2. Analytic and simulated BER of OFDM-IM for $n = 4$ and (a) $M = 4$ and (b) $M = 16$



(a) (b)

Fig. 3. Analytic and simulated BER of OFDM-IM for $n = 8$ and (a) $M = 4$ and (b) $M = 16$

increases. Therefore, for all r, B_{IC} tends to vanish at high SNR. The dominant part of (12) is the second term, B_C. It directly proportional with P_b and the number of active subcarriers exposed to P_b. For given n and M, B_C may be minimized by decreasing the density of active subcarriers within the subblock. Also, for a given r, large n values are to be avoided because they produce large sets of SAPs which increases $(1 - P_{IC})$ and consequently increases B_C. Finally, it can be stated that for better overall average BER performance, then smaller k, n, and M values are preferred to be used in OFDM-IM with ME detection.

V. Simulation Results

An OFDM-IM system with ME detector is simulated using Matlab in order to check the validity of the derived BER expression. The OFDM-IM block consists of 128 subcarriers grouped into subblocks of n subcarriers. The simulated system is tested for $n = 4$ and $n = 8$, $r = 1/n$, 0.5, $(n-1)/n$, and $M = 4$ and $M = 16$, over a Rayleigh fading channel. Figs. 2 and 3 show the tight agreement between the derived and simulated BER performance at different system parameter values. The plots emphasize the statement that a better BER performance is obtained at lower r, n, and M values.

VI. Conclusion

In this paper, the effect of subcarrier activation ratio, r, subblock size, n and QAM symbol modulation order, M, on energy saving, spectral efficiency and BER performance of an OFDM-IM system is studied. It is shown that greater ES is achieved at lower r and M values, while generally the reverse is correct for SE_{IM} with exceptions according to (4). An interesting result from the developed condition in (4) is that for certain (r, n, M) combinations SE_{IM} can be improved to be equal or even greater than SE_{OFDM} by scarifying more energy but with the energy saving advantage still not lost. Then, an approximate BER expression is derived for OFDM-IM exploiting ME detection working over Rayleigh fading channel. It is shown that the BER performance degrades as r approaches unity (OFDM system) and vice versa. This emphasizes the BER advantage of OFDM-IM over OFDM. Finally, the derived expressions and numerical results in this work present a clear view of the joint effect of r, n, and M on ES, SE and BER to help researchers and OFDM-IM system designers to expect these effects and then select suitable values to better meet system design requirements and constraints.

References

[1] E. Basar, U. Aygolu, E. Panayirci, and H. V. Poor, "Orthogonal frequency division multiplexing with index modulation," *IEEE Trans. Signal Process.*, vol. 61, no. 22, pp. 5536 - 5549, Nov. 2013.
[2] M. Wen, X. Cheng, and L. Yang, "Optimizing the energy efficiency of OFDM with index modulation," *in Proc. IEEE Int. Conf. Commun. Workshops*, Nov. 2014, pp. 3135.
[3] P. K. Frenger and N. A. B. Svensson, "Parallel combinatory OFDM signaling," *IEEE Trans. Commun.*, vol. 47, no. 4, pp. 558-567, 1999.
[4] R. Abualhiga and H. Haas, "Subcarrier-index modulation OFDM, " *in Proc. IEEE Int. Symp. Pers., Indoor Mobile Radio Commun.*, 2009, pp. 13-16.
[5] D. Tsonev, S. Sinanovic, and H. Haas, "Enhanced subcarrier index modulation (SIM) OFDM," *in Proc. IEEE GLOBECOM Workshops*, 2011, pp. 728-732.
[6] Y. Ko, "A tight upper bound on bit error rate of joint OFDM and multi-carrier index keying," *IEEE Commun. Lett.*, vol. 18, no.10, pp. 1763-1766, Oct. 2014.
[7] Yue Xiao, Shunshun Wang, Lilin Dan, Xia Lei, Ping Yang and Wei Xiang, "OFDM with interleaved subcarrier-index modulation, " *IEEE Commun. Lett.*, vol. 18, no. 8, pp. 1447-1450, Aug. 2014.
[8] Xiang Cheng, Miaowen Wen, Liuqing Yang, and Yuke Li, "Index modulated OFDM with interleaved grouping for V2X communications," *in Proc. IEEE 17th Int. Conf. ITSC*, 2014, pp.1097-1104.
[9] M. Wen, X. Cheng, M. Ma, B. Jiao, and H. V. Poor, "On the achievable rate of OFDM with index modulation," *IEEE Trans. Signal Process.*, vol. 64,no. 4, pp. 1919-1932, Apr. 2016.
[10] W. Li, H. Zhao, C. Zhang, L. Zhao, and R. Wang, "Generalized selecting sub-carrier modulation scheme in OFDM systems," *in Proc. IEEE Int. Conf. Commun. Workshops*, 2014, pp. 907-911.
[11] E. Basar, "OFDM with index modulation using coordinate interleaving," *IEEE Wireless Commun. Lett.*, vol. 4, no. 8, pp. 381-384, Aug. 2015.
[12] R. Fan, Y. Yu, and Y. Guan, "Generalization of orthogonal frequency division multiplexing with index modulation," *IEEE Trans. Wireless Commun.*, vol. 14, no. 10, pp. 5350-5359, Oct. 2015.
[13] B. Zheng, F. Chen, M. Wen, F. Ji, H. Yu, and Y. Liu, "Low-complexity ML detector and performance analysis for OFDM with in-phase/quadrature index modulation," *IEEE Commun. Lett.*, vol. 19, no. 11, pp. 1893-1896, Nov. 2015.
[14] J. Crawford and Y. Ko, "Low complexity greedy detection method with generalized multicarrier index keying OFDM," *in Proc. IEEE Pers., Indoor., Mobile Radio Commun.*, 2015, pp. 688-693.
[15] A. I. Siddiq, "Low complexity OFDM-IM detector by encoding all possible subcarrier activation patterns," *IEEE Commun. Lett.*,vol. 20, no. 3, pp. 446-449, Mar. 2016.
[16] E. Chatziantoniou, J. Crawford, and Y. Ko, "Performance analysis of a low complexity detector for MCIK-OFDM over TWDP fading," *IEEE Commun. Lett.*,vol. 20, no. 1, pp. 1251-1254, Jun. 2016.
[17] E. Basar, "On Multiple-Input Multiple-Output OFDM with Index Modulation for Next Generation Wireless Networks," *IEEE Trans. Signal Process.*,vol. 64, no. 15, pp. 3868-3878, Aug. 2016.
[18] E. Basar, "Multiple-input multiple-Output OFDM with index modulation," *IEEE Signal Process. Lett.*,vol. 22, no. 12, pp. 2259-2263, Dec. 2015.
[19] M.-S. Alouini and A. Goldsmith, "A unified approach for calculating error rates of linearly modulated signals over generalized fading channels," *IEEE Trans. on Commun.*,vol. 47, no. 9, pp. 1324-1334, Sep. 1999.